

Relative Efficiency of Charges and Quantity Controls in Fisheries with Continuous Stock Growth and Periodically Fixed Instrument Levels

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Abstract *This article presents a simple combination discrete-time/continuous-time model that incorporates continuous population dynamics and fishing activity together with periodic, rather than continuous, instrument adjustment into the decision process for choosing the optimal type and level of regulatory instrument. A per-unit tax and an allocated instantaneous harvest rate quota each drive the system along different time paths, and each results in a different present value of the stream of net benefits generated by harvesting the resource. The choice of instruments is fishery specific; it depends on the parameter values of the fishery in question.*

Introduction

In theory, regulation can potentially improve economic efficiency in markets that exhibit externalities. A wide variety of basic regulatory instruments is available for this purpose, including taxes, allocated quotas, price controls, input controls, and combinations of two or more basic instruments. The question of which type of instrument is superior has been argued in the literature for some time.

Consensus has been reached on the point that in an economically near-perfect world, marred only by the presence of externalities, properly designed instruments of many types all rank equally on the basis of economic efficiency (e.g., see Weitzman 1974; Laffont 1977; Yohe 1978; Dasgupta and Heal 1979; Brown and Boontherawara 1982). There may be distributional, political, administrative, or enforcement considerations that swing the balance in favor of one instrument or another, but in terms of maximizing the sum of consumer and producer surpluses, none can be proved superior.

This conclusion follows from the fact that when perfect information is available, the optimal rate of production, or of effluent discharge, etc., is known. If there are no constraints on the levels at which regulatory instruments can be set, any correctly designed instrument can be set at the level that elicits this rate.

The literature initiated by Weitzman, however, demonstrates that when relatively simple regulatory instruments must be used in the face of "imperfections" other than externalities, such as uncertainty, one instrument or combination of instruments may outperform the others. Which one performs best depends on the assumptions and parameter values of the model being used to describe a particular activity.

Another imperfection in our economic world is the presence of significant costs of: (1) assessing the physical and economic environments of a regulated activity, and (2) adjusting instrument levels as may be warranted by changes in these environments. Because of such costs, instrument levels are often not adjusted as frequently as conditions change. For example, when the size of a renewable natural resource stock changes continuously, the optimal level of whatever instrument is used also changes continuously. Yet instrument levels are adjusted only at periodic intervals. This paper considers the question of optimal instrument choice when instrument levels are periodically fixed while stock size is varying continuously.

The existence of instrument adjustment constraints affects the relative performance of regulatory instruments in dynamic settings. This is due to the fact that different fixed instruments drive the system along different time paths, and hence yield different present values of the stream of net benefits.

Clark (1980) notes that a tax or transferrable quota that is fixed for the duration of the fishing season is suboptimal, and briefly discusses the optimal level of the fixed tax. The fact that adjustment constraints have differential efficiency effects across instruments is mentioned in passing by Dasgupta and Heal (1979), and is alluded to by Rosenman (1986) and by Rosenman and Whiteman (1987). However, this fact has otherwise been largely ignored in the literature on optimal instrument choice. And to date, not many researchers have attempted to apply the methods of the optimal instrument choice literature to apply the methods of the optimal instrument choice literature to regulation of fisheries. There have been a few, including Beddington and May (1977), Andersen (1982), Koenig (1984), and Anderson (1987). But all of these papers were concerned with the effect of *uncertainty* on relative performance of instruments. Anderson's model included periodically fixed instrument levels, but only so that he could analyze the effect of uncertainty when the fishery system is in a temporary steady state. He did not consider the effect on relative performance of the adjustment constraints themselves.

Most previous studies of optimal fishery regulation, with or without uncertainty, have used models and dynamic optimization methods that essentially consider time to be either strictly discrete or strictly continuous. Neither approach, used alone, is capable of dealing with the adjustment constraints problem. On one hand, continuous-time analyses assume continuous monitoring of stock size and continuous adjustment of the chosen regulatory instrument. Results from discrete-time analyses, on the other hand, are usually limited to fisheries in which no natural stock growth or decline occurs during the fishing season. This structure can be realistically assumed only for fisheries with very short open seasons.

Anderson's study, however, developed a combination discrete-time and continuous-time stochastic fishery model. In this study, I have constructed a similar deterministic model and have used it to incorporate instrument level rigidity into the decision process for choosing the optimal type and level of regulatory instrument. The objectives were: (1) to demonstrate that instrument adjustment constraints have differing economic efficiency effects across instruments, and (2) to determine whether there are any generalizations to be made about the ranking of instruments.

The remainder of this article presents an analytical approach based on assumption of linear marginal benefit, marginal cost, and stock growth rate functions, and with harvesting capital that either is fixed permanently or is perfectly variable. It considers two alternative regulatory instruments: a per-unit tax and an aggregate instantaneous harvest rate quota, which is assumed to be optimally allocated among the individual fishing firms. Expressions are derived for single-period net benefits under each instrument.

Then rules are obtained for setting the instruments at optimal fixed levels when stock size at the beginning of the following period does not depend on the quantity harvested during the current period.

Net benefit expressions for each instrument are compared, and it is shown that neither instrument is consistently superior in this simple one-period model; the ranking of instruments depends on the values of the model parameters. Moreover, the relationship between instrument ranking and parameter values is complex, and it is not possible to identify specific combinations of parameter value ranges in which the choice is clear.

There are two exceptions, or special cases, but both are highly restrictive. The first special case is characterized by (1) a constant price of landed fish, (2) a constant instantaneous marginal harvest cost rate, (3) a year-round open fishing season, and (4) an instrument level that is set once, and then never adjusted. In this case, the tax dominates the quota.

The second special case does not require linear marginal benefit, marginal cost, and growth functions. It is characterized simply by a marginal external cost of fishing that does not depend on the size of the stock. Here again, the tax dominates the quota.

While both special cases are quite implausible, they are helpful aids to understanding the general conclusions of the paper. It is for that reason that they are discussed at some length here.

The Model

Notation

r	The (constant) social discount rate.
z	The length of the open fishing season. It may take any value between zero and 1.
i	The period index. It measures time discretely, and goes from one to infinity ($i = 1, \dots, \infty$).
t	Time measured continuously. It goes from $i - 1$ to i during period i .
X_t	The stock size (harvestable biomass) at instant t .
X_0	The stock size at $t = 0$.
h_t	The instantaneous aggregate harvest rate at instant t .
Q_i	The level of the aggregate instantaneous harvest rate quota during period i .
T_i	The level of the per-unit tax during period i .
$B(h_t)$	The instantaneous rate of accrual of total consumption benefit.
$C(X_t, h_t)$	The instantaneous rate of total harvest cost incurrence.
$\bar{R}_i(X_{i-1}, Q_i)$	The present value of the net benefit stream during period i , discounted to the beginning of the period, when a quota system is used. It is the single-period net benefit function under a quota.
$\bar{R}_i(X_{i-1}, T_i)$	The present value of the net benefit stream during period i , discounted to the beginning of the period when a per-unit tax is used. It is the single-period net benefit function under a tax.

Fishing activity and fish population dynamics take place in continuous time, whereas regulatory behavior takes place in discrete time, i.e., the level of the chosen instrument is assumed to be adjusted only at the beginning of each period. It is assumed that the level of capital is either exogenous and permanently fixed, or endogenous and instantana-

neously variable in either direction. Thus capital does not explicitly appear in the harvest cost function.

Both single-period net benefit functions are obtained as follows:

$$R_i(\cdot) = \int_{t-1}^i e^{-r_t} [B(h_t) - C(X_t, h_t)] dt.$$

They are functions of beginning-of-season stock size, X_{t-1} , and of instrument level because of the effect both of these variables have on the time paths of stock size, X_t , and harvest rate, h_t , during the period.

Basic Functions

In order to make additional progress, it is necessary to assume specific functional forms. The three basic functions of the model are (1) the instantaneous total consumption benefit rate function, (2) the instantaneous total harvest cost rate function, and (3) the instantaneous net stock growth rate function:

$$B(h_t) = b_0 h_t - \frac{b_1}{2} h_t^2, \quad (1)$$

$$C(X_t, h_t) = (c_0 - c_2 X_t) h_t + \frac{c_1}{2} h_t^2, \quad (2)$$

$$dX/dt = f_0 - f_1 X_t - h_t, \quad (3)$$

where $f_0 - f_1 X_t$ is the stock growth rate without fishing. The subscripted b 's, c 's, and f 's are fixed, known parameters, and all parameters are assumed to be nonnegative. Following much of the literature, the total benefit function is quadratic in the harvest rate, h_t , and the total cost function is quadratic in the harvest rate and linear in the stock size, X_t . In order to permit derivation of analytical results, the growth function is linear in both harvest rate and stock size. The form $f_0 - f_1 X_t$ can be viewed as an approximation to the right side of the more familiar dome-shaped growth function.

Quota Regulation

The quota considered here is optimal in every way, except for the constraint on frequency of adjustment. Thus it is an instantaneous harvest rate quota, controlling the extraction rate at every instant and not just the cumulative quantity extracted each period. It is also allocated among the members of the industry, or fleet, in a way that minimizes aggregate cost of any given instantaneous catch rate.

For notational convenience, the following discussion refers to the first period, in which time goes from zero to one, but all expressions are identical in every period. Moreover, the period subscript i is omitted.

Assuming that the individual and aggregate quotas are binding constraints at all times during the fishing season, the instantaneous harvest rate is fixed at the level of the aggregate quota, Q . Employing the fixed quota thus precludes continuous adjustment of the harvest rate, which generally is necessary to achieve a full (unconstrained) optimum.

The present value of the net benefit stream during the first period, discounted to time $t = 0$, is

$$\hat{R}(X_0, Q) = \int_0^z e^{-rt} \left\{ b_0 Q - \frac{b_1}{2} Q^2 - (c_0 - c_2 X_t) Q - \frac{c_1}{2} Q^2 \right\} dt, \quad (4)$$

where z is the time at which the fishing season closes, perhaps by decree, e.g., for the protection of gravid females, or perhaps by natural event, such as the onset of winter weather or the annual departure of the fish. The parameter z can take any value between zero and one, inclusive. The open season and the period both begin at time $t = 0$.

The time path of stock size during the fishing season is governed by the stock growth rate equation:

$$dX/dt = f_0 - f_1 X_t - Q. \quad (5)$$

Solving this differential equation yields

$$X_t = \frac{f_0 - Q}{f_1} (1 - e^{-f_1 t}) + X_0 e^{-f_1 t}. \quad (6)$$

Substituting this expression for X_t into equation (4), and rearranging and integrating, gives the present value under a quota:

$$\hat{R}(X_0, Q) = (A + B X_0) Q - C Q^2, \quad (7)$$

$$\text{where } A = \left\{ b_0 - c_0 + \frac{f_0}{f_1} c_2 \right\} \frac{e^{-rz} - 1}{-r} - \left\{ \frac{f_0}{f_1} c_2 \right\} \frac{e^{-(f_1+r)z} - 1}{-(f_1 + r)},$$

$$B = c_2 \frac{e^{-(f_1+r)z} - 1}{-(f_1 + r)},$$

$$C = \left\{ \frac{c_2}{f_1} \right\} \left(\frac{e^{-rz} - 1}{-r} - \frac{e^{-(f_1+r)z} - 1}{-(f_1 + r)} \right) + \left\{ \frac{b_1 + c_1}{2} \right\} \frac{e^{-rz} - 1}{-r}.$$

Under the assumption of nonnegativity of all model parameters, B and C are also unambiguously nonnegative. A may be negative, but only if $b_0 - c_0$ is sufficiently negative to bring this about.

In some fisheries, effort during the current period may have little effect on the size of the harvestable biomass at the beginning of the following period, assuming that the stock is not completely depleted. For example, species with density-independent recruitment and very high fishing or natural mortality rates in the recruited population support single-cohort fisheries. Species that may approximately fit this model include the blue crab, *Callinectes sapidus*, (Richkus 1980), and the American lobster, *Homarus americanus*, (Richardson and Gates 1986).

At any point in time during the fishing season, however, fishing prior to that point always affects the size of the stock available at each instant in the remainder of the season, as shown by equation (6). In such fisheries, the objective of the manager, after observing the size of the stock at the beginning of each period, is to set the quota at Q^* ,

the level that maximizes $\hat{R}(X_0, Q)$, provided that this policy does not result in extinction of the stock. To find Q^* , partially differentiate $\hat{R}(\cdot)$ with respect to Q and set it equal to zero:

$$\hat{R}_Q(X_0, Q^*) = A + BX_0 - 2CQ^* = 0, \quad (8)$$

where $R_Q(\cdot)$ represents the partial derivative of R with respect to Q .

Solving for Q^* gives

$$Q^* = \frac{A + BX_0}{2C}. \quad (9)$$

In a world with perfectly adjustable instruments, optimal quota management would involve continuously adjusting the level of the quota as the size of the fish stock varies throughout the fishing season, with the correct level of the quota at each instant being determined by the principles of dynamic optimization in continuous time. In a world of only periodically adjustable quotas, the fixed (for one period) quota in equation (9) represents a constrained optimal alternative, where Q^* is a kind of average of all the levels an optimally set adjustable quota would take during the course of the period.

Note that using a fixed quota set at Q^* is not necessarily the second-best management alternative. It is possible under some conditions that no regulation at all would be preferable to regulating with a fixed quota. This possibility is ignored in the remainder of the paper.

Equation 9 gives the correct expression for Q^* only under the assumption that the feasible range for optimal instantaneous harvest rate is not bounded. Note especially that for some combinations of the b , c , and f parameter values, we may have to admit the feasibility of negative harvest rates. In such cases, of course, this is an unrealistic assumption because implementing a negative optimal instantaneous harvest rate will not always be practical. The constrained optimal harvest rate would then be zero. However, a bounded range for optimal instantaneous harvest rate is difficult to deal with analytically, although it poses no particular problem in numerical modeling.

It is also necessary to assume that the quota is always binding, even though in fact, the unregulated equilibrium harvest rate might at times be less than the decreed harvest rate. This would be the case when stock size is so low it makes fishing insufficiently profitable to attract all of the needed effort.

Substituting Q^* into equation (7) gives the maximum possible present value obtainable from a quota, by setting the quota at its optimal level:

$$\hat{R}^*(X_0) = \hat{R}(X_0, Q^*) = \frac{(A + BX_0)^2}{4C}. \quad (10)$$

The second order condition for a maximum to exist is $-2C < 0$, or $C > 0$. As noted above, this condition does hold, given the assumption of nonnegative values for all model parameters and provided that the effect of stock size on fishing cost, c_2 , is not zero.

Again as noted above, the term B is unambiguously nonnegative. However, A may take either sign, giving rise to the possibility that the optimal quota could be negative.

This would be the case when fishing is prohibitively expensive at small stock sizes, i.e., $b_0 - c_0$ is negative, and when initial stock size is low.

Tax Regulation

The discussion in this section again refers to the first period for notational convenience, and the period subscript i is again omitted. The analysis here and in the following section applies only when the slopes of the instantaneous marginal benefit and cost functions are not both zero.

Equilibrium instantaneous harvest rate is determined by equating the inverse demand function with the marginal harvest cost function plus the tax, T , and solving:

$$h_t = \frac{b_0 - c_0 + c_2 X_t - T}{b_1 + c_1}, \quad b_1 + c_1 \neq 0. \quad (11)$$

Thus, whereas the harvest rate is not fixed under tax regulation, as it is under a quota, it will not follow the fully optimal time path. In general, the fully optimal path can be followed only when the tax is continuously adjusted.

The present value of the net benefit stream in the first period is

$$\tilde{R}(X_0, T) = \int_0^z e^{-\pi t} \left\{ b_0 h_t - \frac{b_1}{2} h_t^2 - (c_0 - c_2 X_t) h_t - \frac{c_1}{2} h_t^2 \right\} dt, \quad (12)$$

with X_t and h_t both being functions of X_0 and T , as shown below.

Substituting equation (11) for h_t in the stock growth rate function, equation (3), gives

$$dX/dt = \frac{D}{-(b_1 + c_1)} + \frac{E}{-(b_1 + c_1)} X_t - \frac{T}{-(b_1 + c_1)}, \quad (13)$$

where $D = b_0 - c_0 - (b_1 + c_1)f_0$,

$$E = c_2 + (b_1 + c_1)f_1.$$

Solving this differential equation for X_t gives

$$X_t = \frac{D - T}{-E} (1 - e^{-Ft}) + X_0 e^{-Ft}, \quad (14)$$

$$\text{where } F = \frac{E}{b_1 + c_1}.$$

Substituting expressions (11) and (14) for h_t and X_t in equation (12), and rearranging and integrating yields

$$\tilde{R}(X_0, T) = G + JX_0 + KX_0^2 + (L + MX_0)T - NT^2. \quad (15)$$

where

$$G = \left[\left\{ \left(b_0 - c_0 - \frac{D}{E} c_2 \right)^2 \right\} \frac{e^{-rz} - 1}{-r} + 2c_2 \left\{ \left(b_0 - c_0 - \frac{D}{E} c_2 \right) \frac{D}{E} \right\} \frac{e^{-(F+r)z} - 1}{-(F+r)} \right. \\ \left. + c_2^2 \left\{ \left(\frac{D}{E} \right)^2 \right\} \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \right] \frac{1}{2(b_1 + c_1)},$$

$$J = \left[c_2 \left\{ b_0 - c_0 - \frac{D}{E} c_2 \right\} \frac{e^{-(F+r)z} - 1}{-(F+r)} + c_2^2 \left\{ \frac{D}{E} \right\} \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \right] \frac{1}{(b_1 + c_1)},$$

$$K = \{c_2^2\} \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \frac{1}{2(b_1 + c_1)},$$

$$L = \frac{c_2}{E} \left[\left\{ b_0 - c_0 - \frac{D}{E} c_2 \right\} \frac{e^{-rz} - 1}{-r} - \left\{ b_0 - c_0 - 2 \frac{D}{E} c_2 \right\} \frac{e^{-(F+r)z} - 1}{-(F+r)} \right. \\ \left. - \left\{ \frac{D}{E} c_2 \right\} \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \right] \frac{1}{(b_1 + c_1)},$$

$$M = \left\{ \frac{c_2^2}{E} \right\} \left(\frac{e^{-(F+r)z} - 1}{-(F+r)} - \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \right) \frac{1}{(b_1 + c_1)},$$

$$N = \frac{\left[\left\{ 1 - \left(\frac{c_2}{E} \right)^2 \right\} \frac{e^{-rz} - 1}{-r} + \left\{ \left(\frac{c_2}{E} \right)^2 \right\} \left(2 \frac{e^{-(F+r)z} - 1}{-(F+r)} - \frac{e^{-(2F+r)z} - 1}{-(2F+r)} \right) \right]}{2(b_1 + c_1)}.$$

All of the above expressions except J and L can be shown by inspection to be nonnegative. L can be proved nonnegative by another approach, as explained below. J may be positive or negative, but it will be negative only if $b_0 - c_0$ is sufficiently negative.

If the size of the stock at the beginning of the next period is independent of fishing during the current period, the maximization problem has a one-period planning horizon. The objective of the regulating authority is to select the level of T that maximizes $\bar{R}(X_0, T)$; this is accomplished by partially differentiating $\bar{R}(\cdot)$ with respect to T and setting the derivative equal to zero:

$$\tilde{R}_T(X_0, T^*) = L + MX_0 - 2NT^* = 0. \quad (16)$$

Solving for the optimal tax, T^* , gives

$$T^* = \frac{L + MX_0}{2N}. \quad (17)$$

As with quota regulation, perfect adjustability would permit the desired continuous adjustment of the tax level throughout the fishing season. When instruments are only periodically adjustable, the optimal fixed tax given by equation (17) represents a kind of average of all the levels an optimally set adjustable tax would take during the course of the period. Here again, as in equation (9), the above expression (equation (17)) for the optimal fixed level of a regulatory instrument is correct only under the assumption that

the feasible range for optimal instantaneous harvest rate is not bounded. Note that, unlike the fixed quota, Q^* , T^* is a true second-best alternative to its perfectly adjustable counterpart. This is because the fixed tax can be set at zero when no regulation is preferred to constrained regulation.

Substituting equation (17) into equation (15) gives the maximum possible present value of net benefits under a tax:

$$\tilde{R}^*(X_0) = \tilde{R}(X_0, T^*) = G + JX_0 + KX_0^2 + \frac{(L + MX_0)^2}{4N}. \quad (18)$$

The second order condition for a maximum in T to exist is $-2N < 0$, which in turn requires that N be positive. This condition is met under the assumption that all model parameters are positive.

The term L is difficult to sign by inspection, except when $b_0 - c_0 = 0$, when it is clearly positive, but it can be indirectly signed by referring to equation (17). The optimal tax will never be negative, since the inherent tendency in unregulated fisheries is always toward excessive effort, which must be restrained, not encouraged. Since this must be true even if X_0 is zero, L must be nonnegative.

Tax Versus Quota Regulation with a One-Period Planning Horizon

Neither of the two fixed instruments will drive the fishery system along the fully optimal time path, and each will drive it along a different suboptimal time path. Thus the choice of instrument is a matter of selecting the suboptimal time path that produces the highest present value.

Continuing the assumption that stock size at the beginning of the next period is independent of fishing in the current period, one can make the choice between a tax and a quota system based entirely on their relative performance in a single period. The choice is made by comparing the two instruments, assumed to be set at optimal levels, on the basis of present value of net benefits to be accrued over the course of the upcoming season, given the size of the fish stock at the beginning of the season. The "coefficient of comparative advantage of tax over quota" (the CCA) can be defined, following Weitzman (1974), as

$$\begin{aligned} CCA &= \tilde{R}^*(X_0) - \hat{R}^*(X_0) \\ &= G + JX_0 + KX_0^2 + \frac{(L + MX_0)^2}{4N} - \frac{(A + BX_0)^2}{4C}. \end{aligned} \quad (19)$$

If the CCA were positive, a tax would be superior, and if the CCA were negative, a quota system should be chosen. The sign of $G + JX_0 + KX_0^2$ must be nonnegative, since this expression represents the present value of net benefits when the tax is set at zero, and net benefits can never be negative, even when the tax is set at a suboptimal level. The sign of $G + JX_0 + KX_0^2$ can also be obtained by inspection, since it can be shown to be the integral of a squared expression. Thus if there were conditions under which $(L + MX_0)^2/4N$ is always larger than $(A + BX_0)^2/4C$, then the superiority of the

tax would be guaranteed under those conditions. However, no such conditions can be readily identified, and in general there can be no presumption about the sign of the CCA.

Expression (19) is quadratic in X_0 . Written in standard quadratic form, it is

$$CCA = \left(G + \frac{L^2}{4N} + \frac{A^2}{4C} \right) + \left(J + \frac{LM}{2N} - \frac{AB}{2C} \right) X_0 + \left(K + \frac{M^2}{4N} - \frac{B^2}{4C} \right) X_0^2. \quad (20)$$

If all three coefficients of this quadratic were of the same sign, the CCA would also have the same sign for all positive X_0 . Again, however, the signs cannot be determined without specific values for the parameters of the model.

By differentiating the CCA with respect to each of the various parameters of the model, one could hope to gain some idea of the effect that changes in the parameter values would have on relative performance of the two instruments. Unfortunately, however, the resulting expressions for the partial derivatives cannot be signed either, without specific parameter values.

The First Special Case

There is a special case in which the CCA can be signed when only some of the parameter values are specified. This is the case of (1) a constant price of fish, (2) a constant instantaneous marginal harvest cost, (3) a year-round open fishing season, and (4) an instrument level that is set once, and then never adjusted. In this case, the tax dominates the quota.

With perfectly elastic demand and supply, and an infinite planning horizon, the fully optimal management program generally calls for driving the stock as rapidly as possible to an optimal steady-state level (Clark and Munro 1975). If the initial stock size is greater than the optimal steady-state level, X^* , all available fishing effort should be applied to the stock until it is reduced to X^* . If initial stock size is less than X^* , there should be no fishing at all until the stock grows to X^* . Then once the stock size reaches X^* , optimal harvest is constant at the rate that maintains the steady state.

Figure 1 explains the optimality of "bang-bang" management graphically. The first order condition for an interior optimum at each instant is $P - \Phi(X_t) = MC(X_t)$, where P is the constant price of harvested fish, Φ is the marginal external cost of extracting fish from the stock, a function of stock size, and MC is the instantaneous marginal harvest cost, which is constant with respect to harvest rate but which varies inversely with stock size. But for all stock sizes other than X^* , $P - \Phi(X)$ and $MC(X)$ are parallel lines, resulting in corner solutions with optimal harvest rates of either zero or the maximum feasible rate for the current stock size. When the stock size reaches X^* , the two lines coincide, and the optimal harvest rate becomes h^* , which equals the natural rate of stock growth at X^* .

Thus ideally the regulating authority would adjust a quota continuously through time until the stock size reaches X^* . If, as depicted in Figure 1, initial stock size, X_0 , is larger than X^* , the optimal quota program consists of a series of aggregate quota levels at least as large as the maximum feasible harvest rate for each stock size, i.e., nonbinding constraints, until X^* is reached, and then resetting the quota to the binding level h^* . Similarly, a tax set equal to $\Phi(X)$ would vary as stock size declined until the steady state was attained.

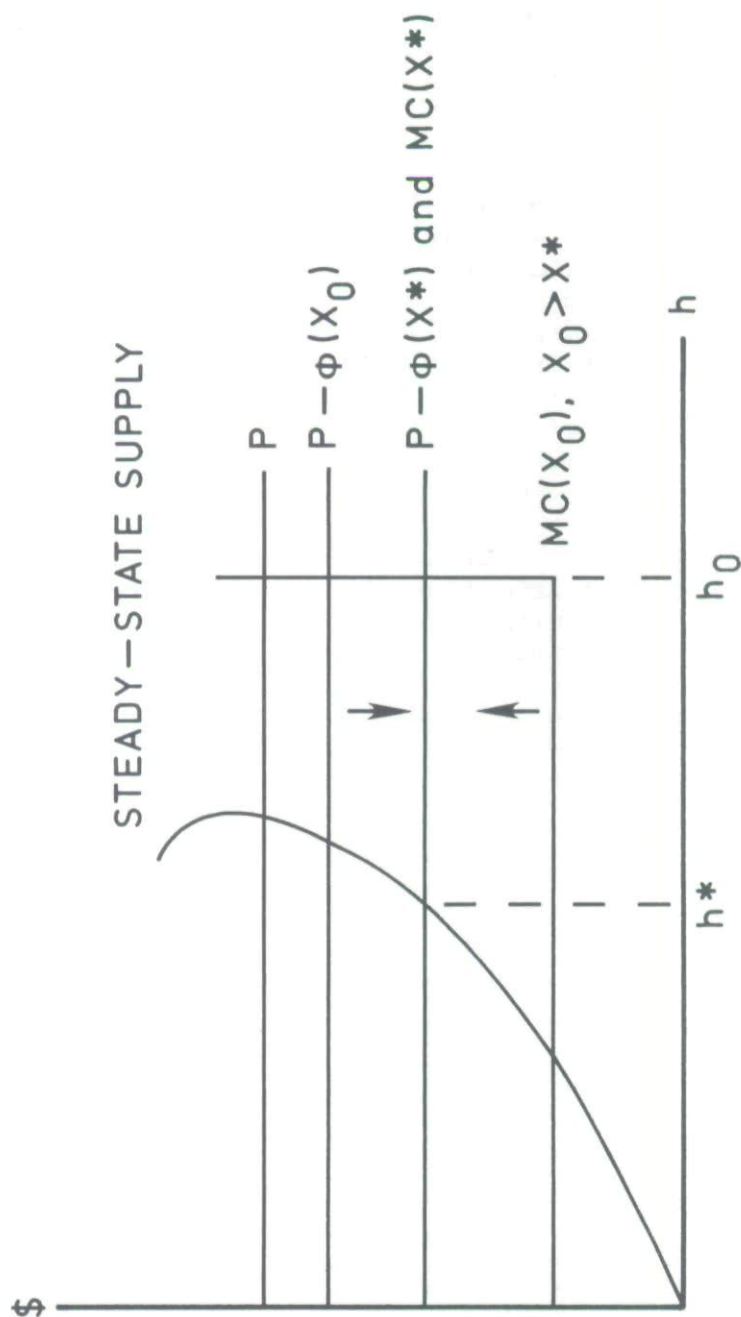


Figure 1. Fishery with perfectly elastic supply and demand.

A quota that is set once and never changed is not able to drive the system along the fully optimal time path. It will take longer for the system to reach a steady state under a fixed quota because it will be a binding constraint on the harvest rate during at least part of the transition to the steady state. Moreover, the optimal level of the fixed quota is not h^* , since Q^* is a kind of average of *all* the values the fully optimal instantaneous harvest rate takes, both before and after the steady state is reached. Therefore, the final steady-state values of all variables, including X , will differ from those of the fully optimal solution.

However, there is a fixed tax level capable of driving the system along the fully optimal time path; adjustment during or after the transition to steady state is not necessary. This tax level is equal to $\Phi(X^*)$, the value that the marginal external cost will take when the optimal steady state has been reached. As long as stock size is greater than X^* , the marginal harvest cost line will be lower than the price net of the tax, $P - \Phi(X^*)$, and fishermen will voluntarily commit all available effort to the fishery. When stock size equals X^* , the marginal cost line coincides with the net price line, and h^* is the equilibrium harvest rate.

Obviously, the fact that a full optimum can be achieved with a fixed tax, but not with a fixed quota, implies that the tax is the superior instrument in this special case. The argument for the superiority of the tax follows symmetrical lines when initial stock size is less than X^* .

To prove tax superiority mathematically, it is first necessary to determine the limits, as $b_1 + c_1$ approaches zero and as z approaches infinity, of the terms **A**, **B**, and **C** in equation (7) (single-period net benefit under quota), and terms **G**, **J**, **K**, **L**, **M**, and **N** in equation (15), the single-period net benefit under tax.

Throughout the proof that follows, it is assumed that $b_0 = c_0$ in order to reduce the size and complexity of the resulting expressions, but this assumption is not necessary for the conclusion of tax superiority over quota in the one-period model.

Let $y = b_1 + c_1$. By l'Hospital's Rule, the limits of the quota terms are found to be:

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} A = \frac{c_2 f_0}{r(f_1 + r)},$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} B = \frac{c_2}{f_1 + r},$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} C = \frac{c_2}{r(f_1 + r)}.$$

By similar procedures, the limits of the tax terms are:

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} G = 0,$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} J = 0,$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} K = \frac{c_2}{4},$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} L = \frac{f_0}{r},$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} M = \frac{1}{2},$$

$$\lim_{y \rightarrow 0} \lim_{z \rightarrow \infty} N = \frac{4f_1 + 3r}{4rc_2}.$$

As noted above, optimal control theory generally dictates that when the chosen instrument is perfectly adjustable and the planning horizon is sufficiently distant, the optimal time path of the fishery system leads to a steady state. An expression for the optimal steady-state level of the fish stock, designated X^* , is derived in Anderson (1987, equation (18)). It is written as follows for the constant price and marginal cost case:

$$X^* = \frac{f_0}{2f_1 + r}.$$

Equation (10) gives the maximum possible net benefit attainable under the quota. It is written for the constant marginal consumption benefit and harvest cost case by replacing **A**, **B**, and **C** with their respective limits to obtain

$$\hat{R}^*(X_0) = \frac{c_2 f_0^2}{4r(f_1 + r)} + \frac{c_2 f_0}{2(f_1 + r)} X_0 + \frac{rc_2}{4(f_1 + r)} X_0^2.$$

Equation (18) gives the maximum possible net benefit attainable under the tax, by setting the tax according to equation (17). However, equation (17) gives the best fixed tax level only when b_1 and c_1 are not equal to zero. It represents an average of all the levels that an optimally set adjustable tax would take during the period, and this constitutes a second-best optimum, compared with the full optimum that could be attained under an adjustable tax.

But as also noted above, when b_1 and c_1 are both zero optimal control theory prescribes "bang-bang" harvest management, and it is not necessary to set the adjustable tax equal to the marginal external cost at each instant. It is not necessary to vary the tax at all in order to drive stock size along the fully optimal path to the optimal steady-state, X^* . Instead, the tax can be fixed from the beginning at a level equal to the level that the marginal external cost will reach when the system reaches the steady state.

The optimal level of the fixed tax, then, is the marginal external cost of fishing, i.e., the marginal present value of stock, at X^* . An expression for this tax level is derived in Anderson (1987, equation (19)). Here, it is designated T_s^* , for optimal steady-state tax level, and in the zero b_1 and c_1 case it is given by

$$T_s^* = \frac{c_2 f_0}{2f_1 + r}.$$

Thus the correct expression to use for maximum attainable net benefit under tax regulation when price and marginal cost are constant is obtained from equation 15 as

$$\tilde{R}^*(X_0) = \tilde{R}(X_0, T_s^*) = G + JX_0 + KX_0^2 + (L + MX_0)T_s^* - N(T_s^*)^2.$$

Replacing **I**, **J**, **K**, **L**, **M**, and **N** by their respective limits yields

$$\tilde{R}^*(X_0) = \frac{c_2 f_0^2 (4f_1 + r)}{4r(2f_1 + r)^2} + \frac{c_2 f_0}{2(2f_1 + r)} X_0 + \frac{c_2}{4} X_0^2.$$

If the fish stock is already at the optimal steady state when regulation begins ($X_0 = X^*$), the two instruments work equally well:

$$\hat{R}^*(X^*) = \tilde{R}^*(X^*) = \frac{c_2 f_0 (f_1 + r)^2}{r(2f_1 + r)^2}.$$

The first derivatives of the net benefit functions are equal at X^* :

$$\hat{R}_X^*(X^*) = \tilde{R}_X^*(X^*) = \frac{c_2 f_0}{2f_1 + r},$$

which means the two functions are tangent to each other at X^* .

Finally, the second derivatives of the net benefit functions are not equal at X^* :

$$\hat{R}_{XX}^*(X^*) = \frac{c_2 r}{2(f_1 + r)},$$

$$\tilde{R}_{XX}^*(X^*) = \frac{c_2}{2}.$$

The second derivative of $\tilde{R}^*(X_0)$ at X^* is clearly greater than that of $\hat{R}^*(X_0)$, so net benefit under a tax is greater than net benefit under a quota for all initial stock sizes other than the optimal steady-state stock size.

The Second Special Case

When marginal external cost of fishing is constant as stock size changes, then the fully optimal tax level is constant through time. Obviously, a fixed tax, if set at the optimal level, is capable of driving the system along the fully optimal time path. However, the fully optimal harvest rate will still vary continuously, so a fixed quota cannot drive the system in a fully optimal way.

This case may be a mathematical impossibility. It is mentioned here only as an additional expository device to aid in understanding the basic idea of differential efficiency effects of instrument adjustment constraints.

Conclusions

The fundamental conclusions of the foregoing analysis are as follows: (1) different periodically fixed instruments drive the natural resource harvesting system along different time paths, and therefore yield different present values of the net benefit stream, (2) which instrument yields the highest present value depends on the parameter values of the

model, and (3) the structure of the dependence is complex, and it is not possible to generalize about conditions under which one instrument or the other will usually be preferred, except in highly restrictive special cases.

The last conclusion is disappointing because it offers no relatively simple rules to guide fishery managers in their decision making. But then simple rules carry the potential for careless application, and may or may not be beneficial.

These results were obtained from a very simple model in which the size of the stock at the beginning of each period is assumed to be independent of the catch in the previous period. It is a straightforward extension to incorporate the expressions for single-period net benefit into a multiple-period dynamic programming model, dropping the assumption that stock size is independent across periods. However, this exercise yields nothing in the way of additional insight into the instrument choice problem. The present values produced by each instrument in such an analysis depend in even more complex ways on the parameter values, and the relationships again cannot be characterized in general terms.

In order to render the problem analytically tractable, it is necessary even in a one-period model to make rather implausible assumptions regarding the linearity of functions, the variability of capital, the feasibility of negative harvest rates, etc. The analytical approach is not useful, therefore, for actually managing real fisheries, and numerical work with a more realistic model is necessary to put the ideas of this paper to work in a specific fishery. The value of the analytical approach lies simply in demonstrating the differential efficiency effects of adjustment constraints, and their dependence on parameter values.

There is a similarity between the analysis of instrument performance under adjustment constraints and under uncertainty. In both situations, the problem is essentially that it is impossible to set any regulatory instrument at the fully optimal level at all times. Thus the system is going to be driven along a less than fully optimal time path, regardless of which instrument is chosen. The choice of instrument is a matter of finding the one that is the best of a group of suboptimal alternatives.

Finally, when the adjustment interval is not exogenously fixed, as assumed here, the problem of determining the optimal adjustment interval arises. It obviously involves balancing the gains in efficiency realized from more frequent adjustment against the additional costs incurred. But that is a matter beyond the scope of this study.

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